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Comment on a mechanism for dynamical breaking of supersymmetry

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Abstract. We re-examine the so-called Nambu–Jona–Lasinio mechanism suggested by Song, Xu and Chin in breaking the supersymmetry in the Wess–Zumino model and show that this mechanism cannot be justified without assuming special effects between fermions. The fermion condensation suggested by them corresponds to an unstable vacuum configuration and as a result there is no fermion condensation and no supersymmetry breaking in the model they discuss.

1. Introduction

In a recent series of papers, Song, Xu and Chin [1] discussed the possibility that spontaneous supersymmetry breaking can be realized in a chiral symmetric model without adding a Fayet–Iliopoulos or O'Raifeartaigh term. In their analysis the so-called Nambu–Jona–Lasinio (NJL) mechanism was used and they suggest that fermion-pair condensation induces a mass gap between supersymmetric partners. If their mechanism really worked it would open many possibilities in supersymmetric models. The purpose of this paper is to present the shortcomings of their argument and clarify the physical background. The main point is very simple: they neglected the one-loop effects of bosonic particles. Including these contributions correctly, we obtain the well known one-loop effective potential; their solution corresponds to an unstable configuration of this effective potential.

This paper is organized as follows. In section 2 we review the construction of an effective potential in the Wess–Zumino (WZ) model and then we re-examine the so-called NJL method proposed in [1] and clarify the physical background. Concluding remarks are given in section 3.

2. Review of the one-loop effective potential in the WZ model

The analysis of supersymmetry breaking in the WZ model is as old as the modern theory of supersymmetry [2]. Using a superfield method, Fujikawa and Lang [3] constructed a one-loop effective potential for the WZ model and discussed the stability of the supersymmetric vacuum. Many authors, for example [4], later discussed this and related topics.

For notational convention we use the two-component representation: by explicitly separating the vacuum expectation values of bosonic fields we derive the one-loop effective potential by means of the tadpole method [5] instead of by direct evaluation [3].

The starting WZ Lagrangian for a chiral super multiplet is given by

$$L = \Phi^{+}\Phi|_{\theta\theta\bar\theta\bar\theta} + \left[\frac{1}{3!}\lambda\Phi^3|_{\theta\theta} + \tfrac{1}{2}m\Phi^2|_{\theta\theta} + \mathrm{HC}\right]$$
$$= \mathrm{i}\partial_m\bar\psi\bar\sigma^m\psi + \bar A\,\Box\,A + \bar F F + [\tfrac{1}{2}\lambda(A^2F - \psi\psi A) + m(AF - \tfrac{1}{2}\psi\psi) + \mathrm{HC}]. \tag{2.1}$$

Shifting the bose fields of the theory in the fashion

$$\begin{aligned} A &\to A + a \\ F &\to F + f \end{aligned} \tag{2.2}$$

we obtain

$$L' = \mathrm{i}\partial_m\bar\psi\bar\sigma^m\psi + \bar A\,\Box\,A + \bar F F + [\eta(AF - \tfrac{1}{2}\psi\psi) + \tfrac{1}{2}\lambda(AAF - \psi\psi A) + \tfrac{1}{2}\lambda f AA$$
$$+F(ma + \tfrac{1}{2}\lambda a^2 - \bar f) + A\eta f + \mathrm{HC}] \tag{2.3}$$

where

$$\eta = m + \lambda a. \tag{2.4}$$

Before calculating the effective potential we should derive the propagators of the theory. Extracting the quadratic part of the boson fields

$$S_0 = \int \mathrm{d}^4x\,\tfrac{1}{2}\Phi^T A\Phi + \Phi^T J$$
$$\begin{cases} \Phi^T = (A, \bar A, F, \bar F) \\ J = (J, \bar J, K, \bar K) \end{cases}$$
$$A = \begin{pmatrix} -\lambda f & \Box & -\eta & 0 \\ \Box & -\lambda\bar f & 0 & \bar\eta \\ -\eta & 0 & 0 & 1 \\ 0 & -\bar\eta & 10 & \end{pmatrix} \tag{2.5}$$

the matrix A is easily inverted to obtain

$$A^{-1} = \frac{1}{\Delta}\begin{pmatrix} \lambda\bar f & \Box - \bar\eta\eta & \bar\eta(\Box - \bar\eta\eta) & \lambda\bar f f \\ \Box - \bar\eta\eta & \lambda f & \lambda f\bar\eta & \eta(\Box - \bar\eta\eta) \\ \bar\eta(\Box - \bar\eta\eta) & \lambda f\bar\eta & \lambda f\bar\eta\bar\eta & -\lambda^2\bar f f + \Box(\Box - \bar\eta\eta) \\ \lambda\bar f\eta & \eta(\Box - \bar\eta\eta) & -\lambda^2\bar f f + \Box(\Box - \bar\eta\eta) & \lambda\bar f\eta\eta \end{pmatrix} \tag{2.6}$$

where

$$\Delta = (\Box - \bar\eta\eta)^2 - \lambda^2\bar f f. \tag{2.7}$$

The tree-level generating function is now given by

$$\ln Z_0 = -\frac{\mathrm{i}}{2}\int \mathrm{d}^4x(J^T A^{-1} J). \tag{2.8}$$

Looking at $\frac{\delta^2\ln(Z_0)}{\delta J_1\delta J_2}|_{J=0}$ the propagators of the theory are obtained directly.

Now, let us derive the effective potential by means of the tadpole method. According to [5] the following relation exists between the derivative of the effective potential and the 1PI tadpole:

$$\frac{\mathrm{d}V(\phi_0)}{\mathrm{d}\phi_0} = -\Gamma'^{(1)}. \tag{2.9}$$

In this expression ϕ_0 is the vacuum expectation value (VEV) of the field ϕ which can be any scalar field of the theory (in the present theory ϕ is A or F, and ϕ_0 is a or f), and $\Gamma'^{(1)}$ is the 1PI tadpole that is calculated after separating the VEV and quantum fluctuation of the scalar fields as $\phi \to \phi + \phi_0$. Hence, we use (2.3) to calculate $\Gamma'^{(1)}$. Using these relations we obtain

$$\frac{V_0}{\mathrm{d}f} = ma + \frac{\lambda}{2}a^2 - \bar{f} \tag{2.10}$$

and

$$\frac{\mathrm{d}V_1}{\mathrm{d}f} = -\frac{2}{\lambda^2}\int \frac{\mathrm{d}^4 p}{(2\pi)^4}\frac{\bar{f}}{(p^2+\bar{\eta}\eta)^2 - \bar{f}f\lambda^2}. \tag{2.11}$$

After integration we obtain

$$V_0 = \left(ma + \tfrac{1}{2}\lambda a^2\right) f - \bar{f}f + P(\bar{f}, a, \bar{a}) \tag{2.12}$$

and

$$V_1 = \frac{1}{2}\int \frac{\mathrm{d}^4 p}{(2\pi)^4}\ln[(p^2+\bar{\eta}\eta)^2 - \lambda^2\bar{f}f] + H(\bar{a}, a) \tag{2.13}$$

where $P(\bar{f}, a, \bar{a})$ and $H(\bar{a}, a)$ are integration constants. We can impose the supersymmetric boundary conditions

$$V_0|_{f=0} = 0 \tag{2.14}$$

and

$$V_1|_{f=0} = 0. \tag{2.15}$$

Then we recover the effective potential

$$V_0 = [(ma + \tfrac{1}{2}\lambda a^2)f + \mathrm{HC}] - \bar{f}f \tag{2.16}$$

and

$$V_1 = \frac{1}{2}\int \frac{\mathrm{d}^4 p}{(2\pi)^4}\ln\left[1 - \frac{\lambda^2\bar{f}f}{(p^2+\bar{\eta}\eta)^2}\right] \tag{2.17}$$

which is also directly calculated in [3]. The vacuum stability of this potential is well analysed in [3, 4]. Equation (2.17) can be evaluated as

$$V_1 = \frac{\pi^2}{(2\pi)^4}\left\{-\tfrac{1}{2}\lambda^2|f|^2(\ln\Lambda^2 + \tfrac{1}{2}) + \tfrac{1}{2}|f|^2\ln|\eta|^2 + \tfrac{1}{2}|\eta|^2[(1-x^2)\ln(1-x) + (1-x)^2\ln(1+x)]\right\} - (Z-1)|f|^2 \tag{2.18}$$

where we set $x = |\lambda f|/|\eta|^2$, and Λ stands for the ultraviolet cut-off.

We have also added the wavefunction renormalization factor Z (in the last term in (2.18)) in order to absorb the infinity contained in $\log\Lambda^2$. In order to avoid the infrared singularity which can appear when we set $m = 0$ in the next section, we renormalize the wavefunction at

$$|f| = 0 \qquad \text{and} \qquad |\eta| = M \tag{2.19}$$

where M has the dimensions of a mass. The wavefunction renormalization factor is then fixed as

$$Z = 1 - \alpha\left(\ln\frac{\Lambda}{M^2} - 1\right). \tag{2.20}$$

The total effective potential up to one-loop level is now given by

$$V_{\rm eff} = -|f|^2\left(1-\alpha\ln\frac{|\eta|^2}{M^2\lambda^2}\right)+\frac{\alpha|\eta|^4}{2\lambda^2}\left[(1+x)^2\ln(1+x)+(1-x)^2\ln(1-x)-3x^2\right]$$
$$+\lambda\left[(a_1^2-a_2^2)f_1+2a_1a_2f_2\right]+2m(a_1f_1+a_2f_2). \tag{2.21}$$

Here we set

$$\begin{cases} f = f_1 + \mathrm{i}f_2 \\ a = a_1 - \mathrm{i}a_2 \end{cases}$$
$$\alpha = \frac{\pi^2\lambda^2}{2(2\pi)^4} \qquad x = \frac{|\lambda f|}{|\eta|^2}.$$

In order to discuss the vacuum stability, we parametrize f_1 and f_2 by

$$\tan\beta = \frac{f_1}{f_2} \tag{2.22}$$

and evaluate $V_{\rm eff}$ at $\partial V_{\rm eff}/\partial\beta = 0$ (this corresponds to the direction of the valley of the effective potential). We then find

$$V_{\rm eff} = -\frac{|\eta|^4x^2}{\lambda^2}\left(1-\alpha\ln\frac{|\eta|^2}{M^2}\right)+\frac{\alpha}{2}|\eta|^4\left[(1+x)^2\ln(1+x)+(1-x)^2\ln(1-x)-3x^2\right]$$
$$+\frac{x|a||\eta|^2}{\lambda}\sqrt{\lambda^2(a_1^2+a_2^2)+2m\lambda a_1+m^2}. \tag{2.23}$$

To take account of the two possible signs of the square-root we extend the range of x to $-\infty < x < +\infty$. This potential develops an imaginary part for $|x| > 1$ and this means that the solution

$$|f| \neq 0 \qquad \text{and} \qquad |\eta| = 0 \tag{2.24}$$

is dynamically unstable. We can find the stationary value of this effective potential in the region $|x| \leqslant 1$ assuming that α is small. The effective potential can be written as

$$V_{\rm eff} \cong -\frac{|\eta|^4x^2}{\lambda^2}\left(1-\alpha\ln\frac{|\eta|^2}{M^2}\right)+\frac{x|a||\eta|^2}{\lambda}\sqrt{\lambda^2(a_1^2+a_2^2)+2m\lambda a_1+m^2}. \tag{2.25}$$

Taking the minimum of the potential ($\partial V_{\rm eff}/\partial x = 0$) we obtain

$$V_{\rm eff} = \frac{|a|^2}{4}\frac{\lambda^2(a_1^2+a_2^2)+2m\lambda a_1+m^2}{1-\alpha\ln[|\eta|^2/M^2]} \tag{2.26}$$

for

$$x = \frac{1}{2}\frac{\lambda|a|\sqrt{\lambda^2(a_1^2+a_2^2)+2m\lambda a_1+m^2}}{|\eta|^2\left(1-\alpha\ln[|\eta|^2/M^2]\right)}. \tag{2.27}$$

This potential has its minimum at

$$a_1 = 0 \qquad a_2 = 0 \qquad \text{and} \qquad f = 0$$

or

$$a_1 = -\frac{m}{\lambda} \qquad a_2 = 0 \qquad \text{and} \qquad f = 0. \tag{2.28}$$

In both solutions, f is zero and the supersymmetry is not broken. The second solution gives non-zero VEV of a but f still remains zero: the two solutions (2.28) are actually two stable physically equivalent solutions, since one can pass from one to the other by a redefinition of the fields [2]. When we consider the massless WZ model in the next section, the second solution becomes $a_1 = 0, a_2 = 0$ so the VEV of a remains zero. A detailed study of this phenomenon from another point of view is given in [2].

Let us examine the physical meanings of this solution. At the tree level, the equation of motion for the auxiliary field is

$$F = \frac{1}{2}\lambda \bar{A}^2. \tag{2.29}$$

At a first glance this equation seems to suggest that if the tree-level potential develops a non-zero vacuum expectation value $\langle \bar{A} \rangle$ then $\langle F \rangle$ becomes non-zero and the supersymmetry of the theory can be broken spontaneously; but this does not happen. Including higher-order quantum corrections the supersymmetry-breaking vacuum ($\langle F \rangle = \frac{1}{2}\lambda\langle \bar{A}\rangle^2$, $\langle \bar{A} \rangle$ non-zero) becomes unstable and the supersymmetric vacuum ($\langle F \rangle = 0$) remains stable. Furthermore, there is no Λ dependence in the effective potential after renormalization of the wavefunction.

To analyse the behaviour of the effective potential at small $|\eta|$ reliably, the renormalization group improvement of the effective potential has also been discussed in [3]. The effective potential for the massless theory is

$$V_{\text{eff}} = -\frac{|\eta|^4 x^2}{\lambda^2}\left(1 - \alpha \ln \frac{|\eta|^2}{M^2}\right) + \frac{\alpha}{2}|\eta|^4\left[(1+x)^2 \ln(1+x) + (1-x)^2 \ln(1-x) - 3x^2\right] \\ + x|a|^2|\eta|^2. \tag{2.30}$$

The stationary value of this potential in the region $|x| < 1$ is estimated to be

$$V_{\text{eff}} = \frac{\lambda^2 |a|^4}{4\left(1 - \alpha \ln[|\eta|^2/M^2]\right)} \tag{2.31}$$

at

$$x = \frac{\lambda |a|^2}{|\eta|^2\left(1 - \alpha \ln[|\eta|^2/M^2]\right)}. \tag{2.32}$$

Renormalization group improvement of V_{eff} suggests that

$$\begin{aligned} V_{\text{eff}} &\simeq \tfrac{1}{4}(\lambda(M)|a|^3)^{4/3}\lambda(|a|)^{2/3} \\ &\simeq \tfrac{1}{4}(\lambda(M))^2|a|^4 \frac{1}{\left(1 - 3\alpha \ln[|\eta|^2/M^2]\right)^{1/3}} \end{aligned} \tag{2.33}$$

with the running coupling

$$\lambda(|a|) = \frac{\lambda(M)}{\left[1 - 3\alpha \ln[|a|^2/M^2]\right]^{1/2}}. \tag{2.34}$$

Note that the combination $\lambda(M)|a|^3$ is renormalization group invariant in this theory.

V_{eff} in (2.33) has a minimum at $|a| = 0$ for which $\lambda(|a|) \to 0$ and the analysis of V_{eff} is reliable. For $|a| \to 0$, $x \to 0$ in (2.33) and thus $|f| \to 0$ and there is no supersymmetry breaking. This explicit analysis is, of course, consistent with the analysis on the basis of a Witten index [6] and is useful for the discussion in the next section.

We summarize the results restricting ourselves to the massless WZ model. First, there is no supersymmetry-breaking vacuum. Second, the VEV of scalar field A remains zero.

3. The meaning of the NJL method in the WZ model

In this section we re-examine the physical background of the NJL method proposed in [1]. For convenience, we first restate the basic procedure given in [1].

The same Lagrangian (2.1) is used, but at the first stage we eliminate the auxiliary field F using the equation of motion. The result is (with $m = 0$)

$$L = \mathrm{i}\partial_m \bar{\psi}\bar{\sigma}^m \psi + A^* \Box A - [\tfrac{1}{2}\lambda\psi\psi A + \mathrm{HC}] - \tfrac{1}{4}\lambda^2 |A|^4. \tag{3.1}$$

The equations of motion are given by

$$\begin{cases} \Box A + \frac{1}{2}\lambda^2 A^* A A + \frac{1}{2}\lambda\bar{\psi}\bar{\psi} = 0 \\ \Box A^* + \frac{1}{2}\lambda^2 A^* A^* A + \frac{1}{2}\lambda\psi\psi = 0 \\ [\mathrm{i}\partial_m \bar{\sigma}^m - \lambda A]\,\psi = 0. \end{cases} \tag{3.2}$$

Taking the vacuum expectation value of the first equation in (3.2), one obtains

$$\Box\langle A\rangle + \tfrac{1}{2}\lambda^2 \langle AAA^*\rangle = -\tfrac{1}{2}\lambda\langle\bar{\psi}\bar{\psi}\rangle. \tag{3.3}$$

Expansion of $\langle A^*AA\rangle$ and $\langle\bar{\psi}\bar{\psi}\rangle$ to the one-loop level (i.e. to the order of $\hbar$) is given by

$$\begin{cases} \langle A^*AA\rangle = \langle A^*\rangle\langle A\rangle\langle A\rangle + \langle A^*\rangle[\otimes^{AA}] + \langle A\rangle[\otimes^{A^*A}] \\ \langle\bar{\psi}\bar{\psi}\rangle = [\otimes^{\bar{\psi}\bar{\psi}}]. \end{cases} \tag{3.4}$$

Here the results of the one-loop diagrams are symbolically represented. Then, to the one-loop order, equation (3.3) becomes

$$0 = \Box a + \frac{\lambda^2}{2}aaa^* + \frac{\lambda^2}{2}a[\otimes^{A^*A}] + \frac{\lambda^2}{2}a^*[\otimes^{AA}] + \frac{\lambda}{2}[\otimes^{\bar{\psi}\bar{\psi}}]. \tag{3.5}$$

Neglecting the tadpoles of the bosonic fields and setting $\Box a = 0$ in (3.5), we obtain the same answer as in [1]:

$$\lambda aaa^* + \mathrm{Tr}\left[\frac{\lambda}{2}\int \frac{\mathrm{d}^4 p}{(2\pi)^4}\frac{1}{\mathrm{i}\partial_m\bar{\sigma}^m - \lambda a^*}\right] = 0 \tag{3.6}$$

which leads to the fermion-pair condensation and a mass gap between the supersymmetric partners [1]. In fact, equation (3.6) can be rewritten as

$$|a|^2 = 4\int \frac{\mathrm{d}^4 p}{(2\pi)^4}\frac{1}{p^2 - \lambda^2|a|^2}. \tag{3.7}$$

This equation looks like a well known mass-gap equation. The integration requires an ultraviolet cut-off, so the solution (a) of the self-consistent equation (3.7) depends on the ultraviolet cut-off parameter. Shifting the fields in the Lagrangian as $A \to A + a$, with a given by equation (3.7), we obtain the masses

$$m_A^2 = \frac{\lambda^2}{2}|a|^2 \qquad m_\psi = \lambda|a|. \tag{3.8}$$

The supersymmetric partners thus appear to acquire different masses. This is the mechanism noted in [1].

But we must not neglect bosonic tadpoles. As discussed in the previous section, the neglect of bosonic tadpoles in (3.5) is not consistent with the expansion in $\hbar$ and the resulting effective potential corresponds to the expansion around an unstable vacuum (i.e. $x = \frac{1}{2}$ in (2.23)). The meaning of equation (3.3) is now clear: this equation means that the derivative of the effective potential is set to zero at the minimum, i.e. $\frac{\partial(V^0 + V^{\text{one-loop}})}{\partial a^*}|_{\text{vac}} = 0$. One can easily obtain (3.3) by applying the tadpole method (2.9) to the variable a, not to f. Substituting A in (3.1) as $A \to A + a$ and using the tadpole method, one obtains

$$\frac{\mathrm{d}(V_0 + V_1)}{\mathrm{d}a^*} = \frac{\lambda^2}{2}aaa^* + \tfrac{1}{2}\lambda[\otimes^{\bar{\psi}\bar{\psi}}] + \tfrac{1}{2}\lambda^2 a[\otimes^{A^*A}] + \tfrac{1}{2}\lambda^2 a^*[\otimes^{AA}]. \tag{3.9}$$

The evaluation and integration of (3.9) is slightly complicated in the present calculational scheme but the result is the same as (2.30) (see [4]). Of course, there is no cut-off dependence in the final result which explicitly remains in the analysis of [1], nor is there supersymmetry breaking induced by fermion-pair condensation in the full effective potential resulting from (3.9). The stationary point of the effective potential corresponds to the supersymmetry preserving point of (2.30).

In conclusion, we have shown that the supersymmetry breaking solution in [1] is a direct consequence of the neglect of one-loop bosonic effects in the loop expansion of the effective potential. Since no dynamical mechanism is given in [1] to explain why the one-loop fermion effects should be retained and why the one-loop boson effects should be neglected, we conclude that the so-called Nambu–Jona–Lasinio mechanism suggested there is not justified in the conventional framework of field theory without assuming some special attractive force between fermions.

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References

[1] Song H-S and Xu G-N 1993 *J. Phys. A: Math. Gen.* **26** 2699; 1992 *J. Phys. A: Math. Gen.* **25** 4941
Song H-S, Xu G-N and Chin Y A 1993 *J. Phys. A: Math. Gen.* **26** 4463
[2] Iliopoulos J and Zumino B 1974 *Nucl. Phys.* B **76** 310
[3] Fujikawa K and Lang W 1975 *Nucl. Phys.* B **88** 77
[4] Miller R 1984 *Phys. Lett.* **124B** 59; 1984 *Nucl. Phys.* B **241** 535
[5] Weinberg S 1973 *Phys. Rev.* D **7** 2887
[6] Witten E 1982 *Nucl. Phys.* B **202** 253